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Orbifolds of M-Theory and Type II String Theories in Two Dimensions

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ABSTRACT

We consider several orbifold compactifications of M-theory and their corresponding type II duals in two space-time dimensions. In particular, we show that while the orbifold compactification of M-theory on $\mathrm{T}^9/\mathcal{J}_9$ is dual to the orbifold compactification of type IIB string theory on $\mathrm{T}^8/\mathcal{I}_8$, the same orbifold $\mathrm{T}^8/\mathcal{I}_8$ of type IIA string theory is dual to M-theory compactified on a smooth product manifold $\mathrm{K3} \times \mathrm{T}^5$. Similarly, while the orbifold compactification of M-theory on $(\mathrm{K3} \times \mathrm{T}^5)/\sigma \cdot \mathcal{J}_5$ is dual to the orbifold compactification of type IIB string theory on $(\mathrm{K3} \times \mathrm{T}^4)/\sigma \cdot \mathcal{I}_4$, the same orbifold of type IIA string theory is dual to the orbifold $\mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\sigma \cdot \mathcal{J}_1$ of M-theory. The spectrum of various orbifold compactifications of M-theory and type II string theories on both sides are compared giving evidence in favor of these duality conjectures. We also comment on a connection between Dasgupta-Mukhi-Witten conjecture and Dabholkar-Park-Sen conjecture for the six-dimensional orbifold models of type IIB string theory and M-theory.

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1. Introduction:

During last one year or so a large number of dual pairs involving string theories [1–10], M-theory [11–13] and F-theory [14,15] have been constructed by assuming that the orbifolding procedure commutes with duality. In this process, once a dual pair is identified one can obtain another dual pair by further compactifying the theories on an internal manifold $\mathcal{M}$ and then modding out the original pairs by a combined group of discrete symmetries representing an internal symmetry transformation of the original theories as well as any geometric action (s) on the internal manifold. There are, however, notable examples where this procedure breaks down. In fact, all the different cases of constructing dual pairs through orbifolding procedure have been classified by Sen [9]. They fall into three categories depending on whether the geometric action 's' acts freely (without fixed points) on the internal manifold $\mathcal{M}$ or not or whether it acts trivially on $\mathcal{M}$ and has been referred to as categories 1, 2 and 3 respectively. (There are some subtleties involved in this procedure for more general orbifold group $\mathbf{Z}_N$ with $N > 2$ [16–20].) The cases where orbifolding procedure is known not to commute with duality fall into category 3 [9]. It has been shown in [9] through a number of examples in string theory, that this procedure of obtaining dual pairs works for the cases 1 and 2. One of the interesting aspects of this classification is that when the orbifolding procedure falls into categories 1 and 2, it gives sensible dual pairs even when the original dual pairs involve M-theory (if we take M-theory by definition as the strong coupling or the large radius limit of type IIA string theory) and was also emphasized in [12]. This is quite remarkable since this way one can connect various compactifications of M-theory to the known compactifications of string theory and extract certain properties of the former without knowing much about its world volume theory.

In this paper, we study examples of dual pairs involving M-theory and type II string theories in two dimensions which can be obtained from the M-theory definition* (i.e. the equivalence of M-theory compactified on S^1 and type IIA string theory when the radius of the circle goes to zero) through orbifolding procedure of category 2 in the classification made by Sen. It has been conjectured by Dasgupta and Mukhi [22] and also by Sen [12]

*It has been demonstrated beautifully in a recent paper by Sen [21] that all the conjectured duality involving string theories, M- and F-theory compactifications can be 'derived' from a single non-perturbative duality between type I and heterotic string theory in ten dimensions with the gauge group SO(32), T-dualities and the definition of M- and F-theories.

that the two dimensional orbifold model $T^9/\{1, \mathcal{J}_9\}$ of M-theory is equivalent to the two dimensional orbifold model $T^8/\{1, \mathcal{I}_8\}$ of type IIB theory, where $\mathcal{J}_9$ changes the sign of the coordinates of T^9 as well as the sign of the three-form gauge field present in M-theory, whereas $\mathcal{I}_8$ simply changes the sign of the coordinates of T^8. Equivalence of these two models follows from the M-theory definition and through the orbifolding procedure of category 2. By comparing the massless spectrum on both sides, we give further evidence in favor of this conjecture. We show that this two dimensional type IIB model has a gravity multiplet and 8 scalar multiplets as massless spectrum in the untwisted sector. In the twisted sector it has 256 antichiral bosons coming from the 256 fixed points of the orbifold and remain inert under (0, 16) supersymmetry of the model. An identical spectrum has been obtained in the M-theory orbifold model on $T^9/\{1, \mathcal{J}_9\}$ in ref.[22]. It has been noted in ref.[17], that the orbifold $T^8/\{1, \mathcal{I}_8\}$ of type IIB theory can not be smoothed out to any known manifold unlike the orbifold $T^4/\{1, \mathcal{I}_4\}$ which is known to give a smooth manifold K3. The situation is different for type IIA compactification on the same orbifold, namely, $T^8/\{1, \mathcal{I}_8\}$. In this case, we show the equivalence between type IIA model on $T^8/\{1, \mathcal{I}_8\}$ and K3 $\times$ T^4. In fact, this type IIA orbifold model is equivalent to heterotic string theory on T^8, type I theory on T^8, type IIA (or type IIB) theory on K3 $\times$ T^4, M-theory on $T^4 \times T^5/\{1, \mathcal{J}_5\}$ and M-theory on K3 $\times$ T^5. We thus find the equivalence of type IIA theory compactified on two completely different internal spaces namely, one is an orbifold and the other is a smooth product manifold. We also show how the spectrum of type IIA theory on $T^8/\{1, \mathcal{I}_8\}$ can be reproduced from the K3 $\times$ T^4 compactification. In the former case as also shown by Sen [9] there is a tadpole contribution [23,24] since the Euler charactersitic of the internal space $T^8/\{1, \mathcal{I}_8\}$ is non-zero whereas in the latter case there is no tadpole since the Euler characteristic of K3 $\times$ T^4 is zero.

Next, we consider another two dimensional orbifold model (K3 $\times$ T^5)$/\{1, \sigma \cdot \mathcal{J}_5\}$ of M-theory, where $\mathcal{J}_5$ changes the sign of all the coordinates of T^5 and the three-form gauge field in M-theory. We show that this model is equivalent to an orbifold model (K3 $\times$ T^4)$/\{1, \sigma \cdot \mathcal{I}_4\}$ of type IIB string theory, where $\mathcal{I}_4$ simply changes the sign of the coordinates of T^4. This equivalence can also be understood, as before, through orbifolding procedure on the M-theory definition and the orbifolding procedure again falls into category 2 of Sen's classification. The massless spectrum [25] on the M-theory side coming from the untwisted sector in this case consists of a gravity multiplet and 16 matter mul-

tiplets of the chiral N=8 or (0, 8) supersymmetry. There are also 128 antichiral bosons and 64 antichiral Majorana-Weyl (M-W) spin 1/2 fermions in the untwisted sector which remain inert under the supersymmetry. In the twisted sector there are 256 antichiral M-W spin 1/2 fermions coming from the 256 fixed points of the M-theory orbifold model as dictated by the condition of gravitational anomaly [26] cancellation. These fermions also remain inert under supersymmetry. We show how this spectrum can be reproduced in the above mentioned type IIB orbifold model. As in the previous case, we then look at the same orbifold $(\mathrm{K3} \times \mathrm{T}^4)/\{1, \sigma \cdot \mathcal{I}_4\}$ of type IIA string theory. By using the self-duality symmetry of type IIA string theory compactified on T^4 which exchanges $\mathcal{I}_4$ to $(-1)^{F_L}$ [6], this model can be shown to be equivalent to an orbifold model $\mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\{1, \sigma \cdot \mathcal{J}_1\}$ of M-theory. We study the spectrum of these two models and show that they match if we take into account the tadpole contribution of the orbifold model of type IIA theory. This type IIA orbifold model has a gravity multiplet and 40 scalar multiplets of the non-chiral (4, 4) supersymmetry of which 16 scalar multiplets come from the one-loop tadpole contribution of the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector antisymmetric tensor field.

Sen in ref.[27] conjectured that a six dimensional orbifold model of M-theory on $(\mathrm{K3} \times \mathrm{S}^1)/\{1, \mathcal{J}_1 \cdot \sigma\}$ is equivalent to an orientifold/orbifold model of type IIB theory on $\mathrm{K3}/\{1, \Omega \cdot \sigma\} \equiv \mathrm{K3}/\{1, (-1)^{F_L} \cdot \sigma\}$ considered by Dabholkar and Park [28]. We refer this conjecture to be Dabholkar-Park-Sen (DPS) conjecture. Here Ω denotes the world-sheet parity transformation which is a symmetry of the type IIB string theory. The symmetries Ω and $(-1)^{F_L}$ are conjugate to each other by the SL(2, Z) invariance of the type IIB theory in ten dimensions. The spectrum in the two theories match in an interesting way as was shown in ref.[12]. We here comment on how this conjecture can be seen to follow from Dasgupta-Mukhi-Witten [22,29] (DMW) conjecture about the equivalence between M-theory on $\mathrm{T}^5/\{1, \mathcal{J}_5\}$ and type IIB theory on K3 by orbifolding procedure and assuming that it commutes with duality. In this case the orbifolding procedure falls into category 3 of Sen's classification, where the equivalence between the two resulting theories is the weakest. The reason why we get a sensible dual pair can be traced if we start from the equivalence of type IIA theory and M-theory on S^1 with the radius of the circle going to zero i.e. from the M-theory definition. By taking orbifold on both sides of the M-theory definition DPS conjecture can then be shown to follow and in this case the orbifolding procedure falls into category 2 in the classification.

The organization of this paper is as follows. In subsection 2.1 of section 2, we discuss the equivalence of M-theory on $\mathrm{T}^9/\{1, \mathcal{J}_9\}$ and type IIB theory on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$. In subsection 2.2, we show that the same orbifold model of type IIA theory is equivalent to M-theory compactification on smooth product manifold K3 $\times$ T^5. We show that the spectrum in these two theories match after we take into account the tadpole contribution on the orbifold of type IIA side. Next, we consider the equivalence of M-theory on (K3 $\times$ T^5) $/\{1, \sigma\cdot\mathcal{J}_5\}$ and type IIB theory on (K3 $\times$ T^4)$/\{1, \sigma\cdot\mathcal{I}_4\}$ in subsection 3.1 of section 3. In subsection 3.2, we consider the same orbifold model of type IIA theory and show that it is equivalent to M-theory compactification on the orbifold $\mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\{1, \sigma\cdot\mathcal{J}_1\}$. Again the spectrum matches after we take into account the tadpole contribution on the type IIA side. Finally, in section 4, we comment that DPS conjecture between orbifold models of type IIB theory and M-theory in six dimensions can be obtained from DMW conjecture through orbifolding procedure and assuming that it commutes with duality. Discussions and conclusions are presented in section 5.

2. Orbifolds of Two Dimensional Toroidal Compactification of M-Theory and Type II String Theories:

In this section, we study the orbifold models of some two dimensional toroidal compactification of M-theory and their corresponding duals in type II string theories. In the first part, we show that the orbifold model $\mathrm{T}^9/\{1, \mathcal{J}_9\}$ of M-theory is dual to the orbifold model $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ of type IIB string theory. We compare the massless spectrum on both sides giving evidence in favor of this conjecture. In the second part, we show that the same orbifold model $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ of type IIA string theory is equivalent to a series of two dimensional orbifold models of M-theory and various other string theories. Interestingly, this particular orbifold model is shown to be equivalent to type IIA theory compactified on a smooth product manifold K3 $\times$ T^4. We compare the massless spectra in these two models and find that they match in an interesting way.

2.1 M-Theory on $\mathbf{T}^9/\mathcal{J}_9$ and Type IIB Theory on $\mathbf{T}^8/\mathcal{I}_8$:

The duality between the two dimensional models of M-theory and type IIB theory in

the title of this subsection has been conjectured by Dasgupta and Mukhi [22] and also by Sen [12]. Here $\mathcal{J}_9 \equiv \mathcal{J}_1 \cdot \mathcal{I}_8$ is a discrete symmetry in M-theory [30] which changes the sign of all the nine coordinates of T^9 and also the sign of the three-form gauge field present in M-theory. The spectrum for the M-theory model has been obtained in ref.[22] by making use of the condition of the two dimensional gravitational anomaly cancellation. We will show in this subsection how the same spectrum can be reproduced in the type IIB model on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$. We first like to point out that this duality conjecture can be understood from the M-theory definition i.e. we take M-theory compactified on S^1 as equivalent by definition to type IIA string theory when the radius of the circle goes to zero. We now further compactify the theories on T^8 and mod out the M-theory by the symmetry group $\{1, \mathcal{J}_1 \cdot \mathcal{I}_8\} \equiv \{1, \mathcal{J}_9\}$ and the type IIA theory by the corresponding image $\{1, (-1)^{F_L} \cdot \mathcal{I}_8\}$[†]. Since the compact space $\mathrm{T}^9/\{1, \mathcal{J}_9\}$ has the structure of S^1 fibered over $\mathrm{T}^8/\{1, (-1)^{F_L} \cdot \mathcal{I}_8\}$, we get the following equivalence by applying the duality conjecture fiberwise [5]:

$$\begin{aligned} & \text{M theory on } \ \mathrm{T}^5/\{1, \mathcal{J}_9\} \\ \equiv \ & \text{Type IIA theory on } \ \mathrm{T}^8/\{1, (-1)^{F_L} \cdot \mathcal{I}_8\} \end{aligned} \tag{1}$$

Note that the orbifolding procedure falls into category 2 of Sen's classification. Now using an $R \to 1/R$ duality transformation on one of the circles of T^8 we convert the type IIA model to type IIB model on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ where $(-1)^{F_L} \cdot \mathcal{I}_8$ in type IIA theory gets converted[*] to $\mathcal{I}_8$ in type IIB theory and thus 'proves' the duality conjecture proposed in the title of this subsection.

The massless spectrum of the M-theory model on $\mathrm{T}^9/\{1, \mathcal{J}_9\}$ was shown in ref.[22] to consist of apart from a gravity multiplet, eight scalar multiplets of (0, 16) supersymmetry of the model and 128 antichiral bosons which remain inert under the supersymmetry. In order to cancel the gravitational anomaly it was found that one requires 512 antichiral M-W fermions which come from the 512 fixed points of $\mathcal{I}_9$ on T^9 of the model. Thus summarizing the massless spectrum including the untwisted as well as the twisted sector states we have:

$$(g_{\mu\nu},\, \phi,\, 16\psi_\mu^-,\, 16\lambda^+)$$

[†]By looking at the massless spectrum it can be easily checked that the symmetry $\mathcal{J}_1$ in M-theory has the same effect as $(-1)^{F_L}$ in the type IIA theory.

[*]This effect can also be checked by comparing the massless spectrum in both type IIA and type IIB models.

$$\begin{aligned} & 8 \times (16\phi^+,\, 16\lambda^+) \\ & 128\phi^- \qquad \text{and} \qquad 512\lambda^- \end{aligned} \tag{2}$$

Note that the graviton and the gravitino have formally -1 degree of freedom in two dimensions and so, the graviton has to be compensated by a scalar and a gravitino has to be compensated by a spin 1/2 M-W fermion. Here we have generically denoted the scalars by ϕ and M-W spin 1/2 fermions by λ. Also the superscripts $(+,\, -)$ indicate, respectively, the (chiral, antichiral) bosons and fermions. We will reproduce this spectrum in the type IIB theory on $\mathrm{T}^8/\{1,\, \mathcal{I}_8\}$.

The massless spectrum of type IIB string theory in ten dimensions contains a graviton $g_{\mu\nu}$, an antisymmetric tensor field $B^{(1)}_{\mu\nu}$ and a dilaton $\phi^{(1)}$ in the NS-NS sector. In the R-R sector it contains another antisymmetric tensor field $B^{(2)}_{\mu\nu}$, another scalar $\phi^{(2)}$ and a four-form gauge field $A^-_{\mu\nu\rho\sigma}$ whose field strength is antiself-dual. In the NS-R sector it has an antichiral gravitino $\psi^{(1)\,-}_\mu$ and a chiral M-W spin 1/2 fermion $\lambda^{(1)\,+}$. In the R-NS sector also it has one antichiral gravitino $\psi^{(2)\,-}_\mu$ and a chiral M-W spin 1/2 fermion $\lambda^{(2)\,+}$. Let us first consider the bosonic sector. From the ten dimensional graviton we get a graviton in two dimensions (2d) and 36 scalars. All the scalars in 2d will survive the $\mathcal{I}_8$ projection. $B^{(1)}_{\mu\nu}$ gives 28 scalars in 2d. The dilaton $\phi^{(1)}$ and the other scalar in the R-R sector $\phi^{(2)}$ both give one scalar each in 2d. $B^{(2)}_{\mu\nu}$ gives 28 scalars and $A^-_{\mu\nu\rho\sigma}$ gives 70 half scalars or 35 scalars in 2d. Thus we have one graviton $g_{\mu\nu}$, one dilaton ϕ and $(36+28+1+28+35) = 128$ scalars in the bosonic sector. In the fermionic sector each of the negative chirality gravitinos of the ten dimensional type IIB theory gives 8 gravitinos of negative chirality and 64 positive chirality spin 1/2 M-W fermions that survive the $\mathcal{I}_8$ projection. Note that $\mathcal{I}_8$ acts on the spinors of one chirality with $+1$ and on the other chirality with -1. Also from each of the positive chirality M-W spinors in ten dimensions we will get 8 positive chirality M-W spinors in 2d that survive $\mathcal{I}_8$ projection. Thus in the fermionic sector we have 16 gravitinos of negative chirality which shows that we have (0, 16) supersymmetry and $(2\times 64 + 2\times 8) = 144$ positive chirality M-W spinors. One can also calculate the massless states in the twisted sector, but we notice that the condition of gravitational anomaly cancellation dictates in this case the presence of 256 antichiral bosons. The gravitational anomaly [26] assoicated with spin 3/2 field (chiral minus antichiral) is $I_{3/2} = (23/24)p_1$, that of spin 1/2 field is $I_{1/2} = -(1/24)p_1$ and for the chiral minus antichiral boson is $I_s = -(1/24)p_1$, where p_1 is the anomaly polynomial

and the fermions are the complex fermions. So, the spectrum would be anomaly free if it satisfies $I_{3/2} : I_{1/2} : I_s = 1 : -m : (23+m)$, where m is any integer. The spectrum in this case satisfies $I_{3/2} : I_{1/2} : I_s = -8 : 72 : -256 = 1 : -9 : 32 = 1 : -9 : (23+9)$ and so is anomaly free. Since there are 256 fixed points of $\mathcal{I}_8$ on T^8, so each fixed point in this orbifold model of type IIB theory will contribute one antichiral boson. The 256 antichiral bosons can be converted to 512 antichiral M-W fermions by bose-fermi equivalence in two dimensions and thus the spectrum matches precisely with the M-theory model. Note that the orbifold model $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ of type IIB theory can not be smoothed out to any known manifold like $\mathrm{K3} \times \mathrm{K3}'$ (since this will break the supersymmetry by 1/4th instead of 1/2) or $\mathrm{T}^4 \times \mathrm{K3}$ (since this will give a non-chiral theory instead of a chiral theory). So, next we consider type IIA model on the same orbifold $\mathrm{T}^8/\{1, \mathcal{I}_8\}$, where the situation is different.

2.2 Type IIA on $\mathrm{T}^8/\mathcal{I}_8$ and M-Theory on $\mathrm{K3} \times \mathrm{T}^5$:

In the second part of this section, we show that an orbifold model of type IIA string theory on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ is non-perturbatively equivalent to M-theory model on $\mathrm{K3} \times \mathrm{T}^5$ which, in turn, is equivalent to type IIA theory on $\mathrm{K3} \times \mathrm{T}^4$. So, we have an equivalence between the same theory compactified on two different internal spaces, one is an orbifold whereas, the other is a smooth product manifold. Note that unlike in type IIB case, this equivalence is intuitively possible since type IIA theory compactified either on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ or on $\mathrm{K3} \times \mathrm{T}^4$ gives non-chiral theories in two dimensions with (8, 8) supersymmetry. The equivalence of these two models can be understood in several ways. First, if we start from the M-theory definition, then,

$$
\begin{aligned}
& \text{Type IIA theory on } \mathrm{T}^8/\{1, \mathcal{I}_8\} \\
\equiv\ & \text{M theory on } \mathrm{T}^8/\{1, \mathcal{I}_8\} \times \mathrm{S}^1
\end{aligned} \tag{3}
$$

Now splitting the eight dimensional torus T^8 as a product of two four dimensional torus $\mathrm{T}^4 \times (\mathrm{T}^4)'$ as well as splitting $\mathcal{I}_8 \equiv \mathcal{I}_4 \cdot \mathcal{I}_4'$ and then using the self-duality symmetry of type IIA string theory on T^4 which changes the geometric symmetry $\mathcal{I}_4$ on T^4 to $(-1)^{F_L}$ we can recast the above M-theory model to the M-theory model on $(\mathrm{T}^4 \times (\mathrm{T}^4)' \times \mathrm{S}^1)/\{1, \mathcal{J}_1 \cdot \mathcal{I}_4'\}$. Note that we have replaced $(-1)^{F_L}$ in type IIA theory by $\mathcal{J}_1$ in M-theory. Thus we arrive

at the equivalence:

$$
\begin{aligned}
& \text{Type IIA theory on } \mathrm{T}^8/\{1,\, \mathcal{I}_8\} \\
\equiv\ & \text{M theory on } \mathrm{T}^4 \times \mathrm{T}^5/\{1,\, \mathcal{J}_5\}
\end{aligned}
\tag{4}
$$

The M-theory model in eq.(4) is equivalent, by DMW conjecture, to type IIB theory on $\mathrm{T}^4 \times$ K3. By T-duality this type IIB model is also equivalent to type IIA theory on $\mathrm{T}^4 \times$ K3 which, in turn, is equivalent to M-theory compactified on K3 $\times\ \mathrm{T}^5$. Thus we have the following chain of equivalences between various models:

$$
\begin{aligned}
& \text{Type IIA theory on } \mathrm{T}^8/\{1,\, \mathcal{I}_8\} \\
\equiv\ & \text{M theory on } \mathrm{T}^4 \times \mathrm{T}^5/\{1,\, \mathcal{J}_5\} \\
\equiv\ & \text{Type IIB theory on } \mathrm{T}^4 \times \mathrm{K3} \\
\equiv\ & \text{Type IIA theory on } \mathrm{T}^4 \times \mathrm{K3} \\
\equiv\ & \text{M theory on } \mathrm{K3} \times \mathrm{T}^5
\end{aligned}
\tag{5}
$$

We can also understand the equivalence of the models mentioned in the title of this subsection if we start from the equivalence of type IIB model on $\mathrm{T}^8/\{1,\, (-1)^{F_L} \cdot \mathcal{I}_8\}$ and type IIB model on $\mathrm{T}^8/\{1,\, \Omega \cdot \mathcal{I}_8\}$, where Ω is the world-sheet parity invariance of the type IIB theory. These models were shown to be equivalent by Sen [9] as $(-1)^{F_L}$ gets precisely converted to Ω by the SL(2, Z) invariance of the type IIB string theory in ten dimensions. The first theory is equivalent to type IIA theory on $\mathrm{T}^8/\{1,\, \mathcal{I}_8\}$ and the second theory is equivalent by $R \to 1/R$ duality transformation on all the circles of T^8 to type IIB theory on $\mathrm{T}^8/\{1,\, \Omega\}$, which is nothing but type I theory on T^8. By the ten dimensional string-string duality of type I and heterotic string theory with gauge group SO(32), we get the second model to be equivalent to heterotic string theory on T^8. By the six dimensional string-string duality between heterotic string on T^4 and type IIA theory on K3, we get the model to be equivalent to type IIA theory on K3 $\times\ \mathrm{T}^4$ which, in turn, is equivalent to M-theory on K3 $\times\ \mathrm{T}^5$. We thus have the following chain of dualities:

$$
\begin{aligned}
& \text{Type IIA theory on } \mathrm{T}^8/\{1,\, \mathcal{I}_8\} \\
\equiv\ & \text{Type I theory on } \mathrm{T}^8 \\
\equiv\ & \text{Heterotic string theory on } \mathrm{T}^8 \\
\equiv\ & \text{Type IIA theory on } \mathrm{K3} \times \mathrm{T}^4 \\
\equiv\ & \text{M theory on } \mathrm{K3} \times \mathrm{T}^5
\end{aligned}
\tag{6}
$$

We now show that the massless spectra in the two different type IIA models on $T^8/\{1, \mathcal{I}_8\}$ and K3 $\times$ T^4, indeed, are the same. The massless spectrum of type IIA string theory in ten dimensions has a graviton $g_{\mu\nu}$, an antisymmetric tensor field $B_{\mu\nu}$ and a dilaton ϕ in the NS-NS sector while in the R-R sector it has a gauge field A_μ and a three-form antisymmetric tensor field $A_{\mu\nu\rho}$. In the fermionic sector it contains a positive chirality gravitino ψ_μ^+ and a positive chirality M-W spinor λ^+ in the NS-R sector, whereas in the R-NS sector it has a negative chirality gravitino ψ_μ^- and a negative chirality M-W spinor λ^-.

We first consider the orbifold model of type IIA string theory on $T^8/\{1, \mathcal{I}_8\}$. This model has already been studied by Sen in ref.[9]. We will study this model in more detail and from a different point of view. In the bosonic sector of this type IIA reduction, we will get in 2d one graviton $g_{\mu\nu}$ and 36 scalars from the ten dimensional graviton, we also get 28 scalars from $B_{\mu\nu}$ and one scalar from the dilaton. Note that both A_μ and $A_{\mu\nu\rho}$ will not give any scalar in 2d since they change sign under $\mathcal{I}_8$. Thus we have one graviton, one dilaton and $(36 + 28) = 64$ scalars in the bosonic sector. In the fermionic sector, we will get from the ten dimensional gravitino ψ_μ^+, 8 gravitinos of positive chirality ψ_μ^+ in 2d and 64 negative chirality M-W spinors λ^- that survive the $\mathcal{I}_8$ projection. Also from ten dimensional M-W spinor λ^+ we get 8 positive chirality M-W spinors in 2d. In the R-NS sector we get 8 gravitinos of negative chirality and 64 positive chirality M-W spinors in 2d from the ten dimensional gravitino ψ_μ^-. Finally, from λ^- in ten dimensions we get 8 negative chirality M-W spinors in 2d. Thus collecting all the states in the untwisted sector we have one graviton $g_{\mu\nu}$, one dilaton ϕ, 64 scalars and 8 gravitinos of positive chirality ψ_μ^+, 8 gravitinos of negative chirality ψ_μ^- also 72 positive chirality λ^+ and 72 negative chirality λ^- M-W spinors:

$$
\begin{aligned}
&(g_{\mu\nu},\, \phi,\, 8\psi_\mu^+,\, 8\lambda^-,\, 8\psi_\mu^-,\, 8\lambda^+) \\
&8 \times (8\phi^+,\, 8\lambda^+), \qquad \text{and} \qquad 8 \times (8\phi^-,\, 8\lambda^-)
\end{aligned}
\tag{7}
$$

It can be easily checked that there are no massless states in the twisted sector of this theory. The massless states in the twisted sector can only arise in the R-R sector since the left or right moving fermions in the NS sector gives vacuum energy greater than zero. But the R-R sector ground state in this case does not survive GSO projection [9].

In studying the consequences of the six-dimensional string-string duality between type IIA string theory on K3 and heterotic string theory on T^4, it was found by Vafa and Witten

[23] that certain two dimensional compactifications of string theories are inconsistent because of the presence of tadpoles of the NS-NS sector antisymmetric tensor field. In these computations of tadpoles, it was realized later by Sethi, Vafa and Witten [24] that the tadpole contribution is simply proportional to the Euler characteristic χ of the compact eight dimensional manifold divided by 24. It was, therefore, argued that the two dimensional compactification of string theories can be made consistent by introducing $\chi/24$ number of one-branes in the internal space. So, if χ is not divisible by 24 or is negative[†], then the tadpoles can not be removed and the corresponding two-dimensional compactification remains inconsistent.

The two dimensional type IIA compactification we are discussing also contains one-loop tadpole from the NS-NS sector antisymmetric tensor field. The tadpole contribution in this theory can be easily calculated by computing the Euler characteristic of the eight dimensional compact space $\mathrm{T}^8/\{1, \mathcal{I}_8\}$. The result is[*] [31]:

$$\chi(\frac{\mathrm{T}^8}{\mathcal{I}_8}) = (0 - 256)/2 + 2 \times 256 = 384 \tag{8}$$

where $\chi(\mathrm{T}^8) = 0$, 256 is the number of fixed points in this model and 2 is the order of the symmetry group. Thus we find that the tadpoles can be cancelled by introducing $384/24 = 16$ elementary type IIA strings in the internal space as noted also by Sen [9] using different method. The collective coordinates of the 16 type IIA strings with (8, 8) supersymmetry will give 128 bosons, 128 positive chirality and 128 negative chirality fermions as additional massless states. Adding these states in (7), we find the complete massless spectrum of type IIA theory on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ to be:

$$\begin{aligned} &(g_{\mu\nu},\, \phi,\, 8\psi^+_\mu,\, 8\lambda^-,\, 8\psi^-_\mu,\, 8\lambda^+) \\ &24 \times (8\phi^+,\, 8\lambda^+), \qquad \text{and} \qquad 24 \times (8\phi^-,\, 8\lambda^-) \end{aligned} \tag{9}$$

Thus this model has (8, 8) supersymmetry and 24 scalar multiplets alongwith a gravity multiplet.

We will see that the same spectrum can be reproduced by compactification of type IIA theory on K3 $\times$ T^4. Under K3 reduction $g_{\mu\nu}$ gives one graviton and 58 scalars in

[†]When χ is negative one requires anti-branes to cancel the tadpoles. Since anti-branes carry wrong chirality matter multiplets this breaks the supersymmetry of the model but the compactification could be consistent [24].

[*]This should be understood as the Euler characteristic of the smooth manifold formed after blowing up the orbifold as usual.

six dimensions; these, in turn, under T^4 reduction gives one graviton, 10 scalars and 58 scalars (from 58 scalars in 6d) in 2d. Similarly, $B_{\mu\nu}$ gives 28 scalars, 6 from T^4 reduction and 22 from K3 reduction. The dilaton gives one scalar in 2d. The gauge field A_μ gives 4 scalars in 2d (it does not give any scalar under K3 reduction). Finally, the three-form gauge field gives one three-form gauge field and 22 vector gauge fields under K3 reduction. Under T^4 reduction, $A_{\mu\nu\rho}$ gives 4 scalars and the 22 vector gauge fields give 88 scalars in 2d. Thus in the bosonic sector of the K3 $\times$ T^4 reduction of type IIA theory we have ($g_{\mu\nu}$, ϕ, 192 scalars).

In the fermionic sector the ten dimensional positive chirality gravitino gives one gravitino of positive chirality and 20 Weyl spinors of negative chirality in six dimensions under K3 reduction. They under T^4 reduction gives 4 gravitinos of positive and 4 gravitinos of negative chirality. It also gives 32 M-W spinors — 16 each with both positive and negative chiralities. Then from 20 Weyl spinors of negative chirality we get 80 M-W spinors of positive and 80 M-W spinors of negative chirality. Also from a single positive chirality M-W spinor in ten dimensions we get 4 of positive chirality and 4 of negative chirality M-W spinors in 2d. In the R-NS sector the spectrum is exactly the same. So, collecting all the states in the fermionic sector we have $2 \times (16 + 80 + 4) = 200$ positive chirality and 200 negative chirality M-W spinors apart from 8 gravitinos of both positive and negative chiralities. This is precisely the spectrum we have obtained from the type IIA compactification on $T^8/\{1, \mathcal{I}_8\}$. We also like to point out that there is no tadpole contribution of this type IIA model on K3 $\times$ T^4 since the Euler characteristic of this product manifold is zero. Note that the tree-level spectrum of type IIA theory on K3 $\times$ T^4 is identical with the tree-level and one-loop level spectrum of type IIA theory on $T^8/\{1, \mathcal{I}_8\}$, indicating that the two models are non-perturbatively equivalent. For a perturbative T-duality we expect the spectrum to match separately at every perturbative level.

3. Orbifolds of Two Dimensional Compactification of M-Theory and Type II String Theories Involving K3:

In this section, we study some orbifold models of two dimensional compactification of M-theory and type II string theories involving K3. In the first part of this section, we consider an orbifold compactification of M-theory on (K3 $\times$ T^5)$/\{1, \sigma \cdot \mathcal{J}_5\}$ and show that it is equivalent to the orbifold compactification of type IIB string theory on (K3

$\times$ $T^4)/\{1,\, \sigma \cdot \mathcal{I}_4\}$. By comparing the massless spectra of both these models we provide evidence in favor of this duality conjecture. In the second part, as in the previous section, we show that the same orbifold model of type IIA theory is dual to an orbifold compactification of M-theory on $T^4 \times (K3 \times S^1)/\{1,\, \sigma \cdot \mathcal{J}_1\}$. We compute the massless spectra of these two models and show that they match if we include the tadpole contribution on the type IIA side.

3.1 M-Theory on $(K3 \times T^5)/\sigma \cdot \mathcal{J}_5$ and Type IIB Theory on $(K3 \times T^4)/\sigma \cdot \mathcal{I}_4$:

In this subsection, we study a two dimensional orbifold model of M-theory involving K3 and find the corresponding dual model in the type IIB theory. This provides another dual pair between M-theory and type IIB theory and will be shown to give consistent anomaly-free spectrum having chiral (0, 8) supersymmetry. The duality conjecture mentioned in the title of this subsection can again be understood from the M-theory definition. So, as in the previous section, we start with the equivalence of M-theory on S^1 and type IIA theory when the radius of the circle goes to zero. We then further compactify the models on K3 $\times$ T^4 and mod out by the combined group of discrete transformation $\sigma \cdot \mathcal{J}_5 \equiv \sigma \cdot \mathcal{J}_1 \cdot \mathcal{I}_4$ in the M-theory side and the corresponding image $\sigma \cdot (-1)^{F_L} \cdot \mathcal{I}_4$ on the type IIA side. Since $(K3 \times T^5)\ /\{1,\, \sigma \cdot \mathcal{J}_5\}$ has the structure S^1 fibered over $(K3 \times T^4)/\{1,\, \sigma \cdot (-1)^{F_L} \cdot \mathcal{I}_4\}$, by applying duality conjecture fiberwise [5] we get the equivalence:

$$\begin{aligned} & \text{M theory on } (K3 \times T^5)/\{1,\, \sigma \cdot \mathcal{J}_5\} \\ \equiv\; & \text{Type IIA theory on } (K3 \times T^4)/\{1,\, \sigma \cdot (-1)^{F_L} \cdot \mathcal{I}_4\} \end{aligned} \tag{10}$$

Here σ denotes an involution on K3 surface. This involution has been used [32] to construct the type IIA dual of the maximally supersymmetric six dimensional heterotic string compactification of Chaudhuri, Hockney and Lykken [33]. By constructing a special K3 surface it has been shown in [32] that σ changes the sign of the 8 harmonic (1, 1) forms of the K3 surface leaving the other 12 harmonic (1, 1) forms as well as the (0, 0), (0, 2), (2, 0) and (2, 2) forms invariant as a consequence of the Lefschetz fixed point theorem. Thus σ acts on K3 with eight fixed points and exchanges the two E_8 factors in the lattice of the second cohomology elements of K3. We also note that σ does not break the supersymmetry of the model unlike another involution of K3 considered in the literature [2] known as Enriques involution which does not leave the holomorphic (2, 0) form invariant and

breaks the supersymmetry of the model by half. Now, by making an $R \to 1/R$ duality transformation on one of the circles of T^4 on the type IIA side in eq.(10) we convert this model to type IIB model on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma\cdot\mathcal{I}_4\}$, where as before $(-1)^{F_L}\cdot\mathcal{I}_4$ in type IIA theory gets converted to $\mathcal{I}_4$ in type IIB theory. Thus we 'derived' the proposed duality conjecture between the M-theory and the type IIB model from the M-theory definition. Note again that the orbifolding procedure falls into category 2 of Sen's classification.

In order to compare the massless spectrum of these two models, we mention that that the spectrum for the M-theory on $(\mathrm{K3} \times \mathrm{T}^5)\ /\{1,\, \mathcal{J}_5\cdot\sigma\}$ has already been obtained by Kumar and Ray in ref.[25]. It has been found that this model contains a gravity multiplet, 16 scalar multiplets of (0, 8) supersymmetry and also 128 antichiral bosons as well as 64 antichiral M-W spin 1/2 fermions as massless states in the untwisted sector where the latter states remain inert under supersymmetry. In order to cancel the two dimensional gravitational anomaly it was found that the twisted sector of this theory contributes 256 antichiral M-W spin 1/2 fermions from the $32\times 8 = 256$ fixed points. These twisted sector states also remain inert under supersymmetry. So summarizing the spectrum we have,

$$
\begin{aligned}
&(g_{\mu\nu},\, \phi,\, 8\psi_\mu^-,\, 8\lambda^+)\\
&16\times(8\phi^+,\, 8\lambda^+)\\
&(128\phi^-,\, 64\lambda^-)\\
&256\lambda^-
\end{aligned}
\tag{11}
$$

We now see how the same spectrum can be obtained from the type IIB model on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma\cdot\mathcal{I}_4\}$. Since various K3 reductions of type IIB string theory has been performed in detail in ref.[34], we will be brief here.

We first consider the bosonic sector of type IIB string theory. The massless spectrum for the ten dimensional type IIB string theory is given in the previous section. The ten dimensional graviton will give one graviton in 2d, 10 scalars from T^4 reduction which remain invariant under $\mathcal{I}_4$ and 34 scalars from K3 reduction which remain invariant under σ. From $B^{(1)}_{\mu\nu}$ we get six scalars from T^4 and 14 scalars from K3 since 14 out of 22 two-forms remain invariant under σ. From the dilaton $\phi^{(1)}$, we get one scalar in 2d. In the R-R sector, we get one scalar from $\phi^{(2)}$. We also get 20 scalars from $B^{(2)}_{\mu\nu}$, 6 from T^4 reduction and 14 from K3 reduction. Finally, from $A^-_{\mu\nu\rho\sigma}$, we get one scalar in 6d for K3 reduction half of which comes from dualizing $A^-_{\mu\nu\rho\sigma}$ corresponding to (0, 0) form on K3 and another half comes from (2, 2) form on K3. This gives one scalar in 2d. We also get from it 11

self-dual two-forms and 3 anti self-dual two-forms corresponding to 11 anti self-dual two-forms and 3 self-dual two-forms on K3 that remain invariant under σ. These two-forms, in turn, will give $(6\times 11+6\times 3)=84$ half-scalars or 42 scalars under T^4 reduction. Thus altogether we have, one graviton, one dilaton and $(10+34+6+14+1+6+14+1+42)=128$ scalars in the bosonic sector.

In the fermionic sector, we get from $\psi_\mu^{(1)\,-}$ one gravitino of negative chirality on K3 reduction (this is a Weyl spinor in 6d) which, in turn, gives 4 gravitinos of negative chirality and 16 positive chirality M-W spin 1/2 fermions in 2d after $\mathcal{I}_4$ projection. It also gives 48 positive chirality M-W spinors in 2d corresponding to 12 positive chirality Weyl spinors which remain invariant under σ. Finally, we get 32 negative chirality M-W spinors in 2d corresponding to 8 positive chirality Weyl spinors in 6d which change sign under σ. From $\lambda^{(1)\,+}$ we will get 4 positive chirality M-W spinors in 2d. In the R-NS sector $\psi_\mu^{(2)\,-}$ and $\lambda^{(2)\,+}$ give the identical states. So, collecting all the states in the fermionic sector we get 8 gravitinos of negative chirality ψ_μ^-, $2\times(16+48+4)=136$ positive chirality M-W spinors λ^+ and $2\times 32=64$ negative chirality M-W spinors λ^-.

Thus the complete spectrum can be arranged as a gravity multiplet $(g_{\mu\nu},\ \phi,\ 8\psi_\mu^-,\ 8\lambda^+)$, 16 scalar multiplets $(8\phi^+,\ 8\lambda^+)$ and $(128\phi^-,\ 64\lambda^-)$ which remain inert under (0, 8) supersymmetry of the model. This is the identical spectrum for the corresponding M-theory model in the untwisted sector. It can be easily checked that this spectrum is anomalous. In this case we find that for the spectrum to become anomaly free we need 128 antichiral bosons. The spectrum then satisfies $I_{3/2}:I_{1/2}:I_s=-4:36:-128=1:-9:(23+9)$ and is anomaly free. Since we have precisely 128 fixed points in this orbifold model (σ has eight fixed points on K3 and $\mathcal{I}_4$ has sixteen fixed points on T^4), so, each fixed point will contribute an antichiral boson in the spectrum. The twisted sector of the type IIB model on $(\mathrm{K3}\times\mathrm{T}^4)/\{1,\ \sigma\cdot\mathcal{I}_4\}$ has been obtained by Sen in ref.[9] and it was found to contain 128 antichiral bosons as we found from the anomaly cancellation. These 128 antichiral bosons can be converted to 256 negative chirality M-W spin 1/2 fermions by bose-fermi equivalence and thus we find that the spectrum precisely matches with that of the M-theory model as obtained by Kumar and Ray [25].

It is clear in this case that there are no tadpoles in the two dimensional M-theory model on $(\mathrm{K3}\times\mathrm{T}^5)/\{1,\ \sigma\cdot\mathcal{J}_5\}$ or equivalently the type IIA model on $(\mathrm{K3}\times\mathrm{T}^4)/\{1,\ \sigma\cdot(-1)^{F_L}\cdot\mathcal{I}_4\}$. Note here that $(-1)^{F_L}$ does not pose any problem in the calculation since this type IIA model is equivalent to type IIB model on $(\mathrm{K3}\times\mathrm{T}^4)/\ \{1,\ \sigma\cdot\mathcal{I}_4\}$

by an $R \to 1/R$ duality transformation on one of the circles of T^4. Since the type IIB model does not contain $(-1)^{F_L}$ factor, we can use the world-sheet parity invariance underwhich the NS-NS sector antisymmetric tensor field $B^{(1)}_{\mu\nu}$ changes sign and thus the tadpole contribution vanishes [23]. The situation is completely different if we consider the same orbifold compactification, namely, $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\}$ of type IIA theory to which we turn next.

3.2 Type IIA Theory on $(\mathbf{K3} \times \mathbf{T}^4)/\sigma{\cdot}\mathcal{I}_4$ and M-Theory on $\mathbf{T}^4 \times (\mathbf{K3} \times \mathbf{S}^1)/\sigma{\cdot}\mathcal{J}_1$:

The equivalence between the two models mentioned in the title of this subsection can be understood from the M-theory definition if we use the self-duality symmetry [6] of type IIA string theory compactified on T^4. So, we start from the equivalence of M-theory on S^1 and type IIA string theory when the radius of S^1 goes to zero. If we then further compactify both the theories on $\mathrm{K3} \times \mathrm{T}^4$ and mod out by the symmetry $\sigma \cdot \mathcal{I}_4$ on both sides, we have the following equivalence:

$$\begin{aligned} & \text{M theory on } (\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\} \times \mathrm{S}^1 \\ \equiv\ & \text{Type IIA theory on } (\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\} \end{aligned} \tag{12}$$

Now using the self-duality symmetry of type IIA theory on T^4 which exchanges $\mathcal{I}_4$ to $(-1)^{F_L}$ we can convert the above M-theory model to M-theory on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot (-1)^{F_L}\} \times \mathrm{S}^1$ which is nothing but M-theory on $(\mathrm{K3} \times \mathrm{T}^4 \times \mathrm{S}^1)/\{1,\, \sigma \cdot \mathcal{J}_1\}$. Since the orbifold group $\{1,\, \sigma \cdot \mathcal{J}_1\}$ does not act on T^4, we can extract it out to write

$$\begin{aligned} & \text{M theory on } \mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \sigma \cdot \mathcal{J}_1\} \\ \equiv\ & \text{Type IIA theory on } (\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\} \end{aligned} \tag{13}$$

'proving' the proposed duality conjecture. Note that this M-theory model is equivalent, by DPS conjecture, to type IIB theory on $\mathrm{T}^4 \times \mathrm{K3}/\{1,\, \sigma \cdot (-1)^{F_L}\} \equiv \mathrm{T}^4 \times \mathrm{K3}/\{1,\, \sigma \cdot \Omega\}$.

We now compute the massless spectra of both the models. Let us first look at the M-theory orbifold model on $\mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \sigma \cdot \mathcal{J}_1\}$. The six dimensional orbifold model of M-theory on $(\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \sigma \cdot \mathcal{J}_1\}$ has already been studied by Sen in ref.[27]. It has been found that this model contains apart from a gravity multiplet $(g_{\mu\nu},\, \psi^\alpha_\mu,\, A^-_{\mu\nu})$, 8 vector multiplets $(A_\mu,\, \lambda^\alpha)$, one tensor multiplet $(A^+_{\mu\nu},\, \lambda_\alpha,\, \phi)$ and 12 hypermultiplets $(\lambda_\alpha,\, 4\phi)$ of N=1, D=6 supersymmetry algebra in the untwisted sector. In the twisted sector this

models contains 8 M-theory five-branes which support 8 tensor and 8 hypermultiplets and the complete spectrum including the untwisted as well as the twisted sector states have been shown to be free of gravitational anomaly. Here, $A^-_{\mu\nu}(A^+_{\mu\nu})$ denotes the antiself-dual(self-dual) antisymmetric tensor. α is the spinor index for which up(down) indicates antichiral(chiral) Weyl spinors. So, we just have to consider the T^4 reduction. In the bosonic sector, we will get one graviton and 10 scalars in 2d from the six dimensional graviton $g_{\mu\nu}$. From $A^-_{\mu\nu}$ we will get 6 half-scalars or 3 scalars in 2d. $8A_\mu$ from the 8 vector multiplets will give 32 scalars and $9A^+_{\mu\nu}$ from the 9 tensor multiplets will give 54 half-scalars or 27 scalars in 2d. Also, 9 scalars from 9 tensor multiplets in six dimensions will give 9 scalars in 2d and $20 \times 4 = 80$ scalars from 20 hypermultiplets will give 80 scalars in 2d. So, altogether in the bosonic sector we have one $g_{\mu\nu}$, one dilaton ϕ and $(9+3+32+27+9+80) = 160$ scalars after T^4 reduction.

Similarly, in the fermionic sector a six dimensional negative chirality Weyl gravitino will give 4 negative and 4 positive chirality M-W gravitinos in 2d. It will also give 16 positive and 16 negative chirality M-W spin 1/2 fermions in 2d. From 8 negative chirality Weyl spinors of 8 vector multiplets, we will get 32 negative and 32 positive chirality M-W spinors in 2d. Also, from 9 positive chirality Weyl spinors of 9 tensor multiplets in 6d, we will get 36 positive chirality and 36 negative chirality M-W spinors in 2d after T^4 reduction. Finally, from the 20 positive chirality Weyl spinors of 20 hypermultiplets in 6d, we will get 80 positive and 80 negative chirality M-W spin 1/2 fermions in 2d. So, collecting all the states in the fermionic sector we have 4 M-W gravitinos of positive and negative chiralities and $(16 + 32 + 36 + 80) = 164$ M-W spin 1/2 fermions of both chiralities. We can arrange the complete spectrum in a gravity multiplet and 40 scalar multiplets of (4, 4) supesymmetry in 2d as follows,

$$\begin{aligned}
&(g_{\mu\nu},\, \phi,\, 4\psi^+_\mu,\, 4\lambda^-,\, 4\psi^-_\mu,\, \lambda^+)\\
&40 \times (4\phi^+,\, 4\lambda^+) \qquad \text{and} \qquad 40 \times (4\phi^-,\, 4\lambda^-)
\end{aligned} \tag{14}$$

We will see in the following that an identical massless spectrum can be reproduced in the two dimensional orbifold model of type IIA theory on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\}$. The massless spectrum of type IIA string theory in ten dimensions has been given in section 2.2. We here consider the reduction. The ten dimensional graviton will give a six dimensional graviton and 34 scalars after K3 reduction which remain invariant under σ. They, in turn, will give one graviton and 10 scalars for T^4 reduction which remain invariant under $\mathcal{I}_4$. We will

also get 34 scalars in 2d from 34 scalars in 6d. The ten dimensional antisymmetric tensor $B_{\mu\nu}$ will give one antisymmetric tensor and 14 scalars for K3 reduction which remain invariant under σ. In 2d they will give 20 scalars, six of which come from T^4 reduction of $B_{\mu\nu}$ in 6d. The ten dimensional dilaton will give one scalar in 2d. In the R-R sector the vector gauge field will not give any scalar in 2d since it changes sign under $\mathcal{I}_4$. From the three-form gauge field we will get, in six dimensions, one three-form gauge field, 14 vector gauge fields which remain invariant under σ and 8 vector gauge fields which change sign under σ. The three-from gauge field and 14 vector gauge fields will not give any scalar for T^4 reduction since they change sign under $\mathcal{I}_4$. But, from 8 vector gauge fields which change sign for K3 reduction under σ will give 32 scalars for T^4 reduction that again change sign under $\mathcal{I}_4$. So, in the bosonic sector we have altogether one graviton, one dilaton and $(10 + 34 + 6 + 14 + 32) = 96$ scalars in 2d.

Let us next consider the fermionic sector. Type IIA theory in ten dimensions has a positive chirality M-W gravitino and a positive chirality M-W spin 1/2 fermion in the NS-R sector. In the R-NS sector it has a negative chirality gravitino and a negative chirality spin 1/2 M-W fermion. For K3 reduction, the ten dimensional ψ_μ^+ will give one positive chirality Weyl gravitino, 12 negative chirality Weyl spinors that remain invariant under σ and 8 negative chirality Weyl spinors that change sign under σ. The six dimensional Weyl gravitino, in turn, for T^4 reduction will give 4 positive chirality M-W gravitinos and 16 negative chirality M-W spin 1/2 fermions that remain invariant under $\mathcal{I}_4$. From the 12 negative chirality Weyl spinors, we will get 48 negative chirality M-W spin 1/2 fermions for T^4 reduction that remain invariant under $\mathcal{I}_4$. Also, from 8 negative chirality Weyl spinors which changed sign under σ, we will get 32 positive chirality M-W spin 1/2 fermions under T^4 reduction which remain invariant under $\mathcal{I}_4$. Finally, the ten dimensional positive chirality M-W spinor will give 4 positive chirality M-W spin 1/2 fermions in 2d. In the R-NS sector after the reduction, we will get the identical fermionic states with opposite chiralities. So, altogether we have 4 gravitinos of positive and 4 gravitinos of negative chiralities. We also have $(16 + 48 + 32 + 4) = 100$ positive and 100 negative chirality M-W spin 1/2 fermions. Therefore, the states in the untwisted sector can be arranged as a gravity multiplet and 24 scalar multiplets as follows:

$$
\begin{aligned}
&(g_{\mu\nu},\, \phi,\, 4\psi_\mu^+,\, 4\lambda^-,\, 4\psi_\mu^-,\, \lambda^+) \\
&24 \times (4\phi^+,\, 4\lambda^+) \qquad \text{and} \qquad 24 \times (4\phi^-,\, 4\lambda^-)
\end{aligned}
\tag{15}
$$

Comparing this spectrum with (14), we find that we still need 16 more scalar multiplets in the type IIA orbifold model on $(\mathrm{K3} \times \mathrm{T}^4)/\{1, \sigma \cdot \mathcal{I}_4\}$ to match the spectrum with the corresponding M-theory model. It can be easily checked again, as in the type IIA theory on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$, that there are no twisted sector states in this type IIA model since the massless states in the twisted sector can only arise as R-R ground state. But, the R-R ground state in this type IIA model does not survive GSO projection. The additional states, as we will see, come from the one-loop tadpole contribution of the type IIA orbifold model on $(\mathrm{K3} \times \mathrm{T}^4)/\{1, \sigma \cdot \mathcal{I}_4\}$. The tadpole contribution can be easily calculated, as before, by calculating the Euler characteristic of the compact eight dimensional internal space. The Euler characteristic of the smooth manifold obtained by blowing up the orbifold $(\mathrm{K3} \times \mathrm{T}^4)/\{1, \sigma \cdot \mathcal{I}_4\}$ is given as,

$$\begin{aligned} \chi(\frac{\mathrm{K3} \times \mathrm{T}^4}{\mathcal{I}_4 \cdot \sigma}) &= (0 - 128)/2 + 2 \times 128 \\ &= 192 \end{aligned} \tag{16}$$

where the Euler characteristic of $\mathrm{K3} \times \mathrm{T}^4$ is zero, 128 is the number of fixed points of the orbifold model (σ has 8 fixed points on K3 and $\mathcal{I}_4$ has 16 on T^4) and 2 is the order of the symmetry group. Since 192 is divisible by 24, we can make the model consistent by introducing $192/24 = 8$ elementary type IIA strings in the internal space. This result can also be understood as follows. Note that locally the space $(\mathrm{K3}\times\mathrm{T}^4)/\sigma\cdot\mathcal{I}_4$ has the structure of $\mathrm{T}^8/\mathcal{I}_8$ and therefore, the physics in the neighborhood of the fixed points should remain the same in both the models. We have mentioned in section 2.2 that the tadpoles in the type IIA model on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ can be removed by introducing 16 elementary type IIA strings of (8, 8) supersymmetry. So, each fixed point in these type IIA models acts as a source of $-(1/16)$ unit of antisymmetric tensor field charge. Since in the type IIA model on $\mathrm{T}^8/\{1, \mathcal{I}_8\}$ there are 256 fixed points and each string carries a single unit of antisymmetric tensor field charge, the charge can be canceled by introducing 16 strings in the internal space. Now, the orbifold model of type IIA theory we are discussing here has 128 fixed points containing total $128\times(-1/16) = -8$ units of antisymmetric tensor field charge which can be canceled by introducing 8 elementary type IIA strings of (8, 8) supersymmetry in the internal space. The collective coordinates of 8 elementary type IIA strings with (8, 8) supersymmetry will give rise to 64 scalars and 64 positive as well as 64 negative chirality M-W spin 1/2 fermions. They can be arranged as 16 matter multiplets of (4, 4) supersymmetry of the model we are considering. Now, adding these

states in (15), we find that the complete massless spectrum of the type IIA orbifold model on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\}$ is given as,

$$
\begin{aligned}
&(g_{\mu\nu},\, \phi,\, 4\psi^{+}_{\mu},\, 4\lambda^{-},\, 4\psi^{-}_{\mu},\, \lambda^{+}) \\
&40 \times (4\phi^{+},\, 4\lambda^{+}) \qquad \text{and} \qquad 40 \times (4\phi^{-},\, 4\lambda^{-})
\end{aligned}
\tag{17}
$$

This is precisely the spectrum we have obtained for the orbifold compactification of M-theory on $\mathrm{T}^4 \times (\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \sigma \cdot \mathcal{J}_1\}$ and thus provides evidence in favor of the duality conjecture proposed in the title of this subsection.

4. DPS Conjecture from DMW Conjecture:

It has been conjectured by Dasgupta and Mukhi [22] and also independently by Witten [29] that a six dimensional orbifold compactification of M-theory on $\mathrm{T}^5/\{1,\, \mathcal{J}_5\}$ is equivalent to a six dimensional orbifold compactification of type IIB theory on $\mathrm{T}^4/\{1,\, \mathcal{I}_4\} \simeq$ K3. In a different work Sen [27] has conjectured that a six dimensional orbifold model of M-theory on $(\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \mathcal{J}_1 \cdot \sigma\}$ is equivalent to a six dimensional orientifold model studied by Dabholkar and Park [28] of type IIB theory on $\mathrm{K3}/\{1,\, \Omega \cdot \sigma\}$. This orientifold model by ten dimensional SL(2, Z) invariance is equivalent to six dimensional orbifold model of type IIB theory on $\mathrm{K3}/\{1,\, (-1)^{F_L} \cdot \sigma\}$. It is easy to see that both these conjectures follow from the M-theory definition. In fact, it has already been pointed out by Sen [21] how DMW conjecture follows from the M-theory definition. In this section, we will first show how DPS conjecture follows also from the M-theory definition and then point out that it can be obtained from DMW conjecture as well through orbifolding procedure and assuming that the orbifolding procedure in this case commutes with duality.

By definition M-theory compactified on S^1 is equivalent to type IIA theory. We then further compactify both theories on K3 and mod out M-theory by the combined group of discrete symmetries $\{1,\, \mathcal{J}_1 \cdot \sigma\}$ and type IIA theory by the corresponding image $\{1,\, (-1)^{F_L} \cdot \sigma\}$. Since the space $(\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \mathcal{J}_1 \cdot \sigma\}$ has the structure S^1 fibered over $\mathrm{K3}/\{1,\, (-1)^{F_L} \cdot \sigma\}$, by applying duality conjecture fiberwise we get the following equivalence:

$$
\begin{aligned}
&\text{M theory on } \ (\mathrm{K3} \times \mathrm{S}^1)/\{1,\, \mathcal{J}_1 \cdot \sigma\} \\
\equiv\ &\text{Type IIA theory on } \ \mathrm{K3}/\{1,\, (-1)^{F_L} \cdot \sigma\}
\end{aligned}
\tag{18}
$$

Now by going to the orbifold limit of K3 we write the right hand side of (18) as type

IIA theory on $T^4/\{1, \mathcal{I}_4, (-1)^{F_L} \cdot \eta \cdot \mathcal{I}_4, (-1)^{F_L} \cdot \eta\}$, where T^4 denotes the product of four circles with coordinates (X^6, X^7, X^8, X^9) at the self-dual radius and η denotes the operation $(X^6, X^7, X^8, X^9) \to (X^6, X^7, X^8, X^9 + \pi R_9)$, with R_9 denoting the radius of the ninth circle. Also the involution σ on K3 surface is represented as $\eta \cdot \mathcal{I}_4$ on T^4. Note that although $\eta \cdot \mathcal{I}_4$ has 16 fixed points on T^4, the orbifold identifies them pairwise leaving 8 fixed points as σ on K3. It has been shown by Sen in ref.[12] that this type IIA model is equivalent by an $R \to 1/R$ duality transformation on one of the circles other than the ninth circle to type IIB model on the same orbifold i.e. $T^4/\{1, (-1)^{F_L} \cdot \eta \cdot \mathcal{I}_4, \mathcal{I}_4, (-1)^{F_L} \cdot \eta\} \simeq$ K3$/\{1, (-1)^{F_L} \cdot \sigma\}$. The only difference is that the elements of the orbifold group got reshuffled in the type IIB theory. Thus it shows that DPS conjecture indeed follows from the M-theory definition. Note that in this case the orbifolding procedure falls into the category 2 of Sen's classification.

Now we start from DMW conjecture i.e.

$$\begin{aligned} & \text{M theory on } \ \mathrm{T}^5/\{1, \mathcal{J}_5\} \\ \equiv \ & \text{Type IIB theory on } \ \mathrm{K3} \simeq \mathrm{T}^4/\{1, \mathcal{I}_4\} \end{aligned} \tag{19}$$

Then we mod out the type IIB theory on K3 by the symmetry group $\{1, (-1)^{F_L} \cdot \sigma\}$ i.e. by $\{1, (-1)^{F_L} \cdot \eta \cdot \mathcal{I}_4\}$ on $T^4/\{1, \mathcal{I}_4\}$ and M-theory by the corresponding image $\{1, \eta \cdot \mathcal{I}_4\}$. If we assume that the orbifolding procedure commutes with duality in this case we arrive at the following equivalence

$$\begin{aligned} & \text{M theory on } \ \mathrm{T}^5/\{1, \mathcal{J}_5, \eta \cdot \mathcal{I}_4, \mathcal{J}_1 \cdot \eta\} \\ \equiv \ & \text{Type IIB theory on } \ \mathrm{T}^4/\{1, \mathcal{I}_4, (-1)^{F_L} \cdot \eta \cdot \mathcal{I}_4, (-1)^{F_L} \cdot \eta\} \end{aligned} \tag{20}$$

We now trade in the coordinates X^m of T^5 of M-theory in favor of new coordinates Z^m as

$$Z^m = X^m \qquad \text{for} \qquad 6 \leq m \leq 8, \qquad Z^9 = X^9 + \pi R_9/2, \qquad Z^{10} = X^{10} \tag{21}$$

then in terms of the new coordinates we have the left hand side of (20) as M-theory on $(T^4 \times S^1)\ /\{1, \mathcal{J}_5 \cdot \eta, \mathcal{I}_4, \mathcal{J}_1 \cdot \eta\} \simeq (\mathrm{K3} \times S^1)/\{1, \mathcal{J}_1 \cdot \sigma\}$. Thus we have arrived at DPS conjecture starting form DMW conjecture.

Note that in 'deriving' DPS conjecture this way we have directly taken the orbifold projection without further compactifying the theories on another manifold. So, for this case the orbifolding procedure falls into category 3 of Sen's classification, where it was

mentioned that the duality conjecture for the resulting theories is the weakest. In fact, in most of the cases this way of orbifolding does not lead to sensible dual pairs. The reason why it works in this case is because the same duality conjecture, as we have seen, can be obtained from the M-theory definition when the orbifolding procedure falls into category 2.

5. Discussion and Conclusion:

We have studied in this paper several examples of dual pairs involving M-theory and type II string theories in two space-time dimensions. Dual pairs are obtained from the M-theory definition and through orbifolding procedure of category 2 of the classification made by Sen. In particular, we have considered two dimensional orbifolds of both toroidal compactification and compactification involving a single K3 of M-theory and type II string theories. By analyzing the massless spectrum we have provided evidence in favor of the duality conjecture between M-theory on $\mathrm{T}^9/\{1,\, \mathcal{J}_9\}$ and type IIB theory on $\mathrm{T}^8/\{1,\, \mathcal{I}_8\}$. This provides an example of consistent anomaly-free chiral supergravity theory in two dimensions having (0, 16) supersymmetry. We then pointed out that the same orbifold model $\mathrm{T}^8/\{1,\, \mathcal{I}_8\}$ of type IIA theory is inconsistent because of the presence of tadpoles of the NS-NS sector antisymmetric tensor field. This inconsistency can be removed by introducing 16 elementary type IIA strings in the internal space. We have shown that this type IIA orbifold model is dual to M-theory compactified on a smooth product manifold $\mathrm{K3} \times \mathrm{T}^5$ or equivalently to type IIA theory on $\mathrm{K3} \times \mathrm{T}^4$. The analysis of the massless spectrum is much simpler in the latter case since this is a smooth manifold having Euler characteristic zero and we just had to calculate the untwisted sector states. We have shown that the spectrum matches with the type IIA theory on $\mathrm{T}^8/\{1,\, \mathcal{I}_8\}$ if we take into account the additional states required to remove the inconsistency involving tadpoles. Next, we have considered another two dimensional orbifold model involving a single K3. In this case, again by analyzing the spectrum on both sides, we have provided evidence in favor of the duality conjecture between M-theory on $(\mathrm{K3} \times \mathrm{T}^5)/\{1,\, \sigma \cdot \mathcal{J}_5\}$ and type IIB theory on $(\mathrm{K3} \times \mathrm{T}^4)/\{1,\, \sigma \cdot \mathcal{I}_4\}$. In this case, we have an example of anomaly-free chiral supegravity theory in two dimensions having (0, 8) supersymmetry. Although this type IIB orbifold model does not have tadpoles, the same orbifold model of type IIA theory is inconsistent because of the presence of tadpoles. We have shown that the orbifold

model (K3 $\times$ T^4)/$\{1,\, \sigma \cdot \mathcal{I}_4\}$ of type IIA theory is dual to the orbifold model T^4 $\times$ (K3 $\times$ S^1) /$\{1,\, \sigma \cdot \mathcal{J}_1\}$ of M-theory. We have shown that the massless spectrum in these two theories match if we take into account the additional states required to remove the inconsistency involving tadpoles on the type IIA side. Finally, we have pointed out that the DPS conjecture for the six dimensional orbifold models involving type IIB theory and M-theory simply follows from DMW conjecture of the equivalence of M-theory on T^5/$\{1,\, \mathcal{J}_5\}$ and type IIB theory on K3. Our analysis confirms that when the orbifolding procedure falls into category 2 of the classification made by Sen, it leads to sensible dual pairs of the resulting theories even if the original pairs involve M-theory on one side.

We have not considered in this paper the two dimensional orbifold compactifications involving two K3's. For example, it can be easily seen that the orbifold models of M-theory on (K3 $\times$ K3$'$ $\times$ S^1)/$\{1,\, \sigma \cdot \sigma' \cdot \mathcal{J}_1\}$ is dual to type IIB theory on (K3 $\times$ T^4) /$\{1,\, (-1)^{F_L} \cdot \mathcal{I}_4\} \cdot \{1,\, \sigma \cdot \eta \cdot \mathcal{I}_4\}$, where η is as given in section 4. This duality can also be understood from M-theory definition and through orbifolding procedure of category 2. Similarly, it can be seen again from orbifolding procedure of category 2 on the M-theory definition that the orbifold model (K3 $\times$ K3$'$)/$\{1,\, (-1)^{F_L} \cdot \sigma \cdot \sigma'\}$ of type IIB string theory is dual to the orbifold model (K3 $\times$ T^5) /$\{1, \mathcal{J}_5\} \cdot \{1,\, \sigma \cdot \eta \cdot \mathcal{I}_4\}$ of M-theory. Note that in both cases, the orbifold group in the dual model has the structure $\mathbf{Z}_2 \times \mathbf{Z}_2$ and can not be further simplified to a single $\mathbf{Z}_2$. It has been recognized in ref.[17] that for orbifolding group greater than $\mathbf{Z}_2$, there are some ambiguities in the analysis of the spectrum and requires a careful study for establishing the duality conjecture. We leave this problem for a separate investigation.

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